Certainty and Uncertainty in Cap and Trade System or in Carbon Tax for Green Accounting to Decrease Greenhouse gas Emissions

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ABSTRACT

This paper analyzes the price or quantity controls of greenhouse gases in the presence of uncertain costs. The greenhouse gases are a stock pollutant in the environment. Hence, the marginal benefit curve must be relatively flat which indicates to establish the preference of a price control over a quantity control. In the case of permanent shocks, the traditional comparison of marginal benefits versus marginal costs cannot be measured accurately. The choice between quantity and price controls becomes ambiguous and depends upon a more difficult measurement of marginal costs and benefits. The aim of the paper is to impose taxes to reduce greenhouse gas emissions.

JEL. Classification: F64; K23; L16; L24; L65

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1. INTRODUCTION

Finland first enacted carbon tax in 1990 on fuels, and then Norway, Sweden, and Denmark implemented carbon taxes in 1991 and 1992 (Anderson, Skou, Dengse and Pederson, 2000). Germany implemented an ecological tax on heating fuel, gasoline, natural gas, and electricity in 1999 (IEA, 2007a). Japan enacted a tax on heavy polluting vehicles in 2001 but reduced the tax on low-pollution vehicles to encourage the development and purchase of greener vehicles (IEA, 2007b). In 2001 the UK implemented a climate change levy which adds about 15% to the cost of electricity (IEA, 2007c). Hungary introduced a New Environmental Burden Tariff in 2004 which taxes pollution of the soil, air, and water (IEA, 2007d). Recently USA has established *social cost of carbon* (SCC) for analysis of federal regulations (IWGSCC, 2010).

At present there are two classic alternatives for regulating greenhouse gas (GHG) emissions, which are a cap and trade policy, and a carbon tax policy. Cap and trade is a quantity control policy and carbon tax is a price control policy. The cap and trade system provides a price which is a secondary result of regulating the quantity of GHG emissions. On the other hand the carbon tax

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effectively reduces the quantity of GHG emissions which is a secondary result of setting a price. To reduce GHG emissions both methods contribute and in idealized circumstances they seem equivalent. If we think the policies in the economic view we observe that quantity control (in cap and trade policy) is preferable when the marginal benefits from price control (in carbon tax policy) are sharply sloping as compared against the marginal costs, but price control is preferable when marginal benefits are relatively flat and marginal costs are sharply sloping (Newell and Pizer, 2006). Hence we observe that the marginal benefit function is flat but the marginal cost function slopes sharply (Nordhaus, 1994). The six gases; Carbon dioxide (CO_2), methane (CH_4), nitrous oxide (N_2O), sulphurhexafluouride (SF_6), hydrofluourocarbon (HFC) and perfluourocarbon (PFC), together constitutes six GHG emissions. These six gases briefly called carbon dioxide equivalents (CO_2e). The Kyoto Protocol 1997 has created an international market for allowances to emit six greenhouse gases but emphasis on carbon dioxide (CO_2) (Mohajan, 2011b). At present the world carbon trade includes fulfillment markets in the EU, the USA, and New Zealand, representing over 140 billion US dollars in traded value and as much as 5 gigatons of emissions per year (Linacre, Kossoy, and Ambrosi, 2011). To enlarge the world carbon trade with proposed markets in Australia and Japan, the international market is projected to reach magnitudes of $2-3 trillion by 2020 (Lazarowicz, 2009; Calel, 2011)). In post-Kyoto international framework the international carbon markets remain a key component of many countries' carbon policy (EU, 2011).

The air pollution is the recognition that any meaningful climate change policy has to put a price on CO_2 to decrease GHG emissions. Baumol and Oates (1988) proposed that pricing GHG emissions is a fundamental lesson from environmental economics and the theory of externalities. Mohajan (2011a) shows that the optimal environmental tax should be less than the marginal environmental damages. Environmental economists suggest that the absence of a price charge for scarce environmental resources such as clean air leads to air pollution (Chesney, Taschini and Wang, 2011). The effects of GHG emissions are described in detail by Mohajan (2011b).

2. CERTAINTY, TEMPORARY UNCERTAINTY AND PERMANENT UNCERTAINTY ON CARBON TAX

We divide a time horizon emissions into T periods indexed by i =1, 2,…, T. Emissions in each period is denoted by E_i . The Cost, C is an exponential function of emissions and a cost parameter α and can be expressed for period i as follows (Parsonsa and Taschini, 2011):

$$C_i\left(E_i,\alpha_i\right)=e^{E_i-\alpha_i}\ . \tag{1}$$

The partial derivatives of (1) are marginal costs and can be written as;

$$\frac{\partial}{\partial E_i}C_i\left(E_i,\alpha_i\right)=-e^{E_i-\alpha_i}\ . \tag{2}$$

The negative marginal cost indicates that the higher emissions yield lower cost. If we decrease emissions then abatement increases cost. The increase of the parameter E_i changes the cost upward which also steeps the cost curve. As a result both the cost of abatement and the marginal cost of abatement increase. In this case we face uncertainty in the cost. Now we try to find two contrasting specifications of cost uncertainty as follows:

- In first case, the shocks to the cost parameter are completely temporary or transitory. A shock affects the cost parameter in current period but it has no relation on the cost parameter in any future period.
- In the second case, shocks to the cost have a permanent affects. A shock affects the cost parameter in current period and the expected cost in all future periods is also incremented by the same amount.

The first specification of the cost parameter α_i is given by:

$$\alpha_i = \alpha_0 + i\varepsilon + \nu\,\theta_i, \tag{3}$$

where α_0 is the starting cost parameter, ε is the constant expected growth rate in the mean cost parameter, ν is volatility parameter and θ_i are the shocks to the cost parameter which are independent standard normal random variables.

The second specification of the cost parameter α_i is given by:

$$\alpha_i = \alpha_{i-1} + \varepsilon + \nu\,\theta_i. \tag{4}$$

Now we assume that a fixed aggregate emissions constraint is $\overline{E}$. Hence for *T* periods we can write;

$$\sum_{i=1}^{T} E_i \le \overline{E}. \tag{5}$$

A dynamic emissions policy can be obtain by setting emissions in each period on some function of past aggregate emissions and on the current value of the cost parameter. Let the remaining allowed emissions in period *i* is denoted by $\overline{E}_i$. Hence we can write for *i* = 1, 2,…, *T*-1 as follows:

$$\overline{E}_{i+1} = \overline{E}_i - E_i, \tag{6}$$

where $E_i = E_i\left(\overline{E}_i, \alpha_i\right)$. Now we assume that the value function in period *i* is V_i. In the final period
i = *T*, the total cost of remaining emissions to abate can be written as;

$$V_T\left(\overline{E}_T, E_T, \alpha_T\right) \equiv C\left(E_T\left(\overline{E}_T, \alpha_T\right), \alpha_T\right). \tag{7}$$

The value for the cost minimizing emissions level, $E_T^*\left(\overline{E}_T, \alpha_T\right)$ becomes,

$$\min_{E_T} V_T\left(\overline{E}_T, E_T, \alpha_T\right) \tag{8}$$

subject to (5). The solution of the cost function takes the form;

$$E_T^*\left(\overline{E}_T, \alpha_T\right) = \overline{E}_T . \tag{9}$$

Let the optimize value at the final period be V_T^* which is a function of the remaining allowed emissions coming into the period *T* and the realized cost parameter in the period *T* can be written as;

$$\begin{aligned} V_T^*\left(\overline{E}_T, \alpha_T\right) &\equiv V_T^*\left(\overline{E}_T, E^*{}_T\left(\overline{E}_T, \alpha_T\right), \alpha_T\right) \\ &= C\left(E_T^*\left(\overline{E}_T, \alpha_T\right), \alpha_T\right) \\ &= C\left(E_T^*, \alpha_T\right). \end{aligned} \tag{10}$$

The marginal cost of emissions under the pollution policy (10) becomes;

$$p_T^*\left(\overline{E}_T, \alpha_T\right) \equiv -\frac{\partial}{\partial E_T} C\left(E_T^*\left(\overline{E}_T, \alpha_T\right), \alpha_T\right) = C\left(E_T^*\left(\overline{E}_T, \alpha_T\right), \alpha_T\right) \tag{11}$$

where $p_T^*\left(\overline{E}_T, \alpha_T\right)$ represents the price at the period *T* which is the negative of marginal cost. The logarithm of marginal cost for period *i* is defined by;

$$\ln p_i^* = \alpha_i - E_i^* . \tag{12}$$

Now for earlier periods, $1 \le i < T$, the value function is the total cost of current period emissions plus the discounted expectation of the value function (δ_{α_i}) in the subsequent period as follows:

$$V_i\left(\overline{E}_i, E_i, \alpha_i\right) \equiv C\left(E_i\left(\overline{E}_i, \alpha_i\right), \alpha_i\right) + \delta_{\alpha_i}\left(V_{i+1}^*\left(\overline{E}_{i+1}\left(\overline{E}_i, E_i\right), \alpha_{i+1}\right)\right). \tag{13}$$

The cost minimizing emissions $E_i^*\left(\overline{E}_i, \alpha_i\right)$ becomes as follows:

$$\min_{E_i} V_i\left(\overline{E}_i, E_i, \alpha_i\right). \tag{14}$$

In this situation the optimal value function is given by;

$$V_i^*\left(\overline{E}_i,\alpha_i\right) \equiv V_i\left(\overline{E}_i, E_i^*\left(\overline{E}_i,\alpha_i\right),\alpha_i\right), \tag{15}$$

and the marginal cost of emissions is given by;

$$p_i^*\left(\overline{E}_i,\alpha_i\right) \equiv -\frac{\partial}{\partial E_i} C\left(E_i^*\left(\overline{E}_i,\alpha_i\right),\alpha_i\right) = C\left(E_i^*\left(\overline{E}_i,\alpha_i\right),\alpha_i\right). \tag{16}$$

We now analyze the above solutions for different cases as follows:

2.1 Results in the Certainty Case

For convenient we use the backward programming starting from the end period that is we use index j = T, …, 2, 1. In the certainty case we use $\nu = 0$, then the cost parameter follows the dynamics as below (Parsonsa and Taschini, 2011):

$$\alpha_{j-1} = \alpha_j + \varepsilon = \alpha_0 + (T - j + 1)\,\varepsilon, \tag{17}$$

where $i = T - j + 1$. Now in the last period for j = 1, from (9) we get;

$$E_1^*\left(\overline{E}_1,\alpha_1\right) = \overline{E}_1. \tag{18}$$

Hence from (10) and (1) the value function is given by;

$$V_1^*\left(\overline{E}_1,\alpha_1\right) = C\left(\overline{E}_1,\alpha_1\right) = e^{\alpha_1 - E_1^*}. \tag{19}$$

For j = 2 we can write;

$$\begin{aligned} V_2\left(\overline{E}_2, E_2,\alpha_2\right) &= \delta_{\alpha_2}\left[\, C\left(E_2,\alpha_2\right) + e^{-r}\, V_1^*\left(\overline{E}_1\left(\overline{E}_2, E_2\right),\alpha_1\right)\right] \\ &= e^{\alpha_2 - E_2} + e^{-r}\left(e^{\alpha_1 - \left(\overline{E}_2 - E_2\right)}\right) \\ &= e^{\alpha_2 - E_2} + e^{-r}\left(e^{\alpha_2 + \varepsilon - \left(\overline{E}_2 - E_2\right)}\right) \\ &= e^{\alpha_2}\left\{\, e^{-E_2} + e^{-(r-\varepsilon)} e^{-\left(\overline{E}_2 - E_2\right)}\right\}. \end{aligned} \tag{20}$$

For the first order condition in the cost minimizing emissions we get;

$$\frac{\partial V_2}{\partial E_2} = e^{\alpha_2}\left\{-e^{-E_2^*} + e^{-(r-\varepsilon)} e^{-\left(\overline{E}_2 - E_2^*\right)}\right\} = 0,$$

$$-e^{-E_2^*}+e^{-(r-\varepsilon)}e^{-\left(\overline{E}_2-E_2^*\right)}=0,$$

$$e^{-2E_2^*}=e^{-(r-\varepsilon)}e^{-\overline{E}_2},$$

$$E_2^*=\frac{1}{2}\overline{E}_2-\frac{1}{2}(\varepsilon-r). \tag{21}$$

Then the optimized value function can be written as;

$$V_2^*\left(E_2^*,\alpha_2\right)=2e^{\alpha_2-E_2^*}=2e^{\alpha_2-\frac{1}{2}\overline{E}_2+\frac{1}{2}(\varepsilon-r)}. \tag{22}$$

For j = 3 we can write;

$$V_3\left(\overline{E}_3,E_3,\alpha_3\right)=\delta_{\alpha_3}\left[\,C\left(E_3,\alpha_3\right)+e^{-r}\,V_2^*\left(\overline{E}_2\left(\overline{E}_3,E_3\right),\alpha_2\right)\right]$$

$$=e^{\alpha_3-E_3}+e^{-r}\delta_{\alpha_3}\left(2e^{\alpha_2-\frac{\overline{E}_3-E_3-\varepsilon+r}{2}}\right)$$

$$=e^{\alpha_3-E_3}+2e^{-r}\delta_{\alpha_3}\;e^{\alpha_3+\varepsilon-\frac{1}{2}\nu^2+\nu\,\theta_2-\frac{\overline{E}_3-E_3-\varepsilon+r}{2}}$$

$$=e^{\alpha_3-E_3}+2e^{-r}e^{-\frac{\overline{E}_3-E_3-\varepsilon+r}{2}}e^{\alpha_3+\varepsilon}e^{-\frac{1}{2}\nu^2}\left(\delta_{\alpha_3}\;e^{\nu\,\theta_2}\right)$$

$$=e^{\alpha_3}\left\{e^{-E_3}+2e^{-\frac{\overline{E}_3-E_3-3\varepsilon+3r}{2}}\right\}. \tag{23}$$

For the first order condition in the cost minimizing emissions we get;

$$\frac{\partial V_3}{\partial E_3}=e^{\alpha_3}\left\{e^{-E_3^*}+2e^{-\frac{\overline{E}_3-E_3^*-3\varepsilon+3r}{2}}\right\}=0,$$

$$-e^{-E_3^*}+2e^{-\frac{\overline{E}_3-E_3^*-3\varepsilon+3r}{2}}=0,$$

$$E_3^* = \frac{\overline{E}_3 - E_3^* - 3\varepsilon + 3r}{2},$$

$$E_3^* = \frac{1}{3}\overline{E}_3 - \frac{2}{2}(\varepsilon - r). \tag{24}$$

Then the optimized value function can be written as;

$$V_3^*\left(E_3^*, \alpha_3\right) = 3e^{\alpha_3 - E_3^*} = 3e^{\alpha_3 - \frac{1}{3}\overline{E}_3 + \frac{1}{2}(\varepsilon - r)}.$$

For $j = 4$ we can write;

$$V_4\left(\overline{E}_4, E_4, \alpha_4\right) = \delta_{\alpha_4}\left[\, C\left(E_4, \alpha_4\right) + e^{-r}\, V_3^*\left(\overline{E}_3\left(\overline{E}_4, E_4\right), \alpha_3\right)\right]$$

$$= e^{\alpha_4 - E_4} + e^{-r}\delta_{\alpha_4}\left(3e^{\alpha_3 - \frac{\overline{E}_4 - E_4 - 3\varepsilon + 3r}{3}}\right)$$

$$= e^{\alpha_4 - E_4} + 3e^{-r}\delta_{\alpha_4}\, e^{\alpha_4 + \varepsilon - \frac{1}{2}\nu^2 + \nu\,\theta_4 - \frac{\overline{E}_4 - E_4 - 3\varepsilon + 3r}{3}}$$

$$= e^{\alpha_4 - E_4} + 3e^{-r}e^{-\frac{\overline{E}_4 - E_4 - 3\varepsilon + 3r}{3}} e^{\alpha_4 + \varepsilon} e^{-\frac{1}{2}\nu^2}\left(\delta_{\alpha_4}\, e^{\nu\,\theta_3}\right)$$

$$= e^{\alpha_4}\left\{e^{-E_4} + 3e^{-\frac{\overline{E}_4 - E_4 - 6\varepsilon + 6r}{3}}\right\}. \tag{25}$$

For the first order condition in the cost minimizing emissions we get;

$$\frac{\partial V_4}{\partial E_4} = e^{\alpha_4}\left\{e^{-E_4^*} + 3e^{-\frac{\overline{E}_4 - E_4^* - 6\varepsilon + 6r}{3}}\right\} = 0,$$

$$-e^{-E_4^*} + 3e^{-\frac{\overline{E}_4 - E_4^* - 6\varepsilon + 6r}{4}} = 0\text{'}$$

$$E_4^* = \frac{\overline{E}_4 - E_4^* - 6\varepsilon + 6r}{3},$$

$$E_4^* = \frac{1}{4}\overline{E}_4 - \frac{3}{2}(\varepsilon - r). \tag{26}$$

Then the optimized value function can be written as;

$$V_4^*\left(E_4^*,\alpha_4\right)=4e^{\alpha_4-E_4^*}=4e^{\alpha_4-\frac{1}{4}\overline{E}_4+\frac{3}{2}(\varepsilon-r)}. \qquad (27)$$

Proceeding such a way we can write the general form of the optimal policy as follows:

$$E_j^*=\frac{1}{j}\overline{E}_j-\frac{1}{2}(j-1)(\varepsilon-r). \qquad (28)$$

In calendar time *i* for $j=T-i+1$, (28) can be written as;

$$E_i^*=\frac{1}{T-i+1}\overline{E}_i-\frac{1}{2}(T-i)(\varepsilon-r). \qquad (29)$$

The general form of the optimized form can be written as follows:

$$V_j^*\left(E_j^*,\alpha_j\right)=je^{\alpha_j-E_j^*}=je^{\alpha_j-\frac{1}{j}\overline{E}_j+\frac{1}{2}(j-1)(\varepsilon-r)}. \qquad (30)$$

In calendar time *i* for $j=T-i+1$ we can write (30) as;

$$V_i^*\left(E_i^*,\alpha_i\right)=i\,e^{\alpha_i-E_i^*}=\left(T-i+1\right)e^{\alpha_i-\frac{1}{T-i+1}\overline{E}_i+\frac{1}{2}(T-i)(\varepsilon-r)}. \qquad (31)$$

From (12) the logarithm of marginal cost function becomes;

$$\ln p_i^*=\alpha_i-\frac{1}{T-i+1}\overline{E}_i+\frac{1}{2}(T-i)(\varepsilon-r). \qquad (32)$$

In the certainty case we can translate back to an equation that describes emissions in different periods, *i* = 1, 2, . . . , *T*, which is a function of the total allowed emissions $\overline{E}_i$, the total number of periods *T*, the rate of growth in the cost parameter ε and the discount rate *r* . Hence we can write such an equation by (29) as follows:

$$E_i^*\left(\overline{E},T,\varepsilon,r\right)=\frac{1}{T}E-\frac{T}{2}(\varepsilon-r)+i\left(\varepsilon-r\right). \qquad (33)$$

By using (3), equation (32) can be expressed as follows:

$$\ln p_i^*\left(\overline{E},T,\varepsilon,r\right)=\alpha_0+i\varepsilon-\frac{1}{T}\overline{E}+\frac{1}{2}(\varepsilon-r)+ir. \qquad (34)$$

The emissions increase linearly through time at the rate $(\varepsilon - r)$ which is shown in (33). So that the marginal cost of abatement grows at the discount rate *r* which is shown in equation (34). If $\varepsilon \geq r$ and the related cost parameter is growing at a rate greater than the discount rate, then this adjustment leads to reducing the rate of emissions in current period *i*. This indicates that increasing the realized marginal cost at present to preserve allowed emissions for the later periods when the cost parameter is higher. As a result we have to reduce the growth rate in the realized marginal cost to equal the discount rate.

2.2 Results in the Temporary Shock Case

If the per period shock is temporary the cost parameter follows the dynamics as (Parsonsa and Taschini 2011);

$$\alpha_j = \Psi_j + \nu\,\theta_j \text{ where } \Psi_j = \Psi_{j+1} + \varepsilon. \tag{35}$$

In the backward programming for *j* = 1 we have the same result of (18) and (19). For *j* = 2, we get from (13);

$$\begin{aligned} V_2(E_2, E_2, \alpha_2) &= e^{\Psi_2 + \nu\,\theta_2 - E_2} + e^{-r}\delta_{\alpha_2}\left(e^{\alpha_1 - (\bar{E}_2 - E_2)}\right) \\ &= e^{\Psi_2 + \nu\,\theta_2 - E_2} + e^{-r}\delta_{\alpha_2}\left(e^{\Psi_1 + \nu\,\theta_1 - (\bar{E}_2 - E_2)}\right) \\ &= e^{\Psi_2 + \nu\,\theta_2 - E_2} + e^{-r}e^{-(\bar{E}_2 - E_2)}e^{\Psi_2 + \nu}\delta\left(e^{\nu\,\theta_1}\right) \\ &= e^{\Psi_2}\left[e^{\nu\theta_2 - E_2} + e^{-\left(r - \varepsilon - \frac{1}{2}\nu^2\right)}e^{-(\bar{E}_2 - E_2)}\right]. \end{aligned} \tag{36}$$

For the first order condition in the cost minimizing emissions we get;

$$\frac{\partial V_2}{\partial E_2} = e^{\Psi_2}\left[-e^{\nu\theta_2 - E_2^*} + e^{-\left(r - \varepsilon - \frac{1}{2}\nu^2\right)}e^{-(\bar{E}_2 - E_2^*)}\right] = 0,$$

$$-e^{\nu\,\theta_2 - E_2^*} = e^{-\left(r - \varepsilon - \frac{1}{2}\nu^2\right)}e^{-(\bar{E}_2 - E_2^*)},$$

$$\nu\theta_2 - E_2^* = -\left(r - \varepsilon - \frac{1}{2}\nu^2\right) - \bar{E}_2 + E_2^*,$$

$$E_2^* = \frac{1}{2}\bar{E}_2 - \frac{1}{2}(r - \varepsilon) - \frac{1}{4}\nu^2 + \frac{1}{2}\nu\theta_2. \tag{37}$$

Then the optimized value function can be written as;

$$V_2^*\left(\overline{E}_2,\alpha_2\right)=2e^{\alpha_2-E_2^*}=2e^{\alpha_2-\frac{1}{2}\overline{E}_2+\frac{1}{2}(\varepsilon-r)+\frac{1}{4}\nu^2-\frac{1}{2}\nu\,\theta_2}\,. \tag{38}$$

For j = 3 from (13) we have,

$$V_3\left(\overline{E}_3,E_3,\alpha_3\right)=\delta_{\alpha_3}\left[\,C\left(E_3,\alpha_3\right)+e^{-r}\,V_2^*\left(\overline{E}_2\left(\overline{E}_3,E_3\right),\alpha_2\right)\right]$$

$$=e^{\alpha_3-E_3}+e^{-r}\delta_{\alpha_3}\left(2e^{\alpha_2-\frac{\overline{E}_3-E_3-\varepsilon+r-\frac{1}{2}\nu^2+\nu\theta_2}{2}}\right)$$

$$=e^{\Psi_3+\nu\theta_3-E_3}+e^{-r}\delta_{\alpha_3}\left(2e^{\Psi_2+\nu\theta_2-\frac{\overline{E}_3-E_3-\varepsilon+r-\frac{1}{2}\nu^2+\nu\theta_2}{2}}\right)$$

$$=e^{\Psi_3}e^{\nu\theta_3-E_3}+2e^{-r}e^{-\frac{\overline{E}_3-E_3-\varepsilon+r-\frac{1}{2}\nu^2+\nu\theta_2}{2}}e^{\Psi_3+\nu}\delta\left(e^{\nu\theta_2}e^{-\frac{1}{2}\nu\theta_2}\right)$$

$$=e^{\Psi_3}\left[e^{\nu\theta_3-E_3}+2e^{-\frac{\overline{E}_3-E_3-3\varepsilon+3r-\frac{3}{4}\nu^2}{2}}\right]. \tag{39}$$

For the first order condition in the cost minimizing emissions we get;

$$\frac{\partial V_3}{\partial E_3}=e^{\Psi_3}\left[-e^{\nu\theta_3-E_3^*}+e^{-\frac{\overline{E}_3-E_3^*-3\varepsilon+3r-\frac{3}{4}\nu^2}{2}}\right]=0\,,$$

$$e^{\nu\,\theta_3-E_3^*}=e^{-\frac{\overline{E}_3-E_3^*-3\varepsilon+3r-\frac{3}{4}\nu^2}{2}}\,,$$

$$E_3^*-\nu\,\theta_3=\frac{1}{2}\left(\overline{E}_3-E_3^*-3\nu+3r-\frac{3}{4}\nu^2\right),$$

$$E_3^*=\frac{1}{3}\overline{E}_3-\frac{2}{2}(\varepsilon-r)-\frac{1}{4}\nu^2-\frac{2}{3}\nu\,\theta_3\,. \tag{40}$$

Then the optimized value function can be written as;

$$V_3^*\left(\overline{E}_3,\alpha_3\right)=2e^{\alpha_3-E_3^*}=3e^{\alpha_3-\frac{1}{2}\overline{E}_3+(\varepsilon-r)+\frac{1}{4}\nu^2+\frac{2}{3}\nu\,\theta_3}\,. \tag{41}$$

For j = 4 from (13) we have,

$$V_4\left(\overline{E}_4, E_4, \alpha_4\right) = \delta_{\alpha_4}\left[\, C\left(E_4, \alpha_4\right) + e^{-r}\, V_3^*\left(\overline{E}_3\left(\overline{E}_4, E_4\right), \alpha_3\right)\right]$$

$$= e^{\alpha_4 - E_4} + e^{-r}\,\delta_{\alpha_4}\left(3e^{\alpha_3 - \frac{\overline{E}_4 - E_4 - 3\varepsilon + 3r - \frac{3}{4}\nu^2 - 2\nu\theta_3}{3}}\right)$$

$$= e^{\Psi_4 + \nu\theta_4 - E_4} + 3e^{-r}\,\delta_{\alpha_4}\left(e^{\Psi_3 + \nu\theta_3 - \frac{\overline{E}_4 - E_4 - 3\varepsilon + 3r - \frac{3}{4}\nu^2 - 2\nu\theta_3}{3}}\right)$$

$$= e^{\Psi_4} e^{\nu\theta_4 - E_4} + 3e^{-r} e^{-\frac{\overline{E}_4 - E_4 - 3\varepsilon + 3r - \frac{3}{4}\nu^2 - 2\nu\theta_3}{3}} e^{\Psi_4 + \nu}\,\delta\left(e^{\nu\theta_3} e^{-\frac{2}{3}\nu\theta_3}\right)$$

$$= e^{\Psi_4}\left[e^{\nu\theta_4 - E_4} + 3e^{-\frac{\overline{E}_4 - E_4 - 6\varepsilon + 6r - \frac{11}{12}\nu^2}{3}}\right]. \qquad (42)$$

For the first order condition in the cost minimizing emissions we get;

$$\frac{\partial V_3}{\partial E_3} = e^{\Psi_4}\left[-e^{\nu\theta_4 - E_4^*} + e^{-\frac{\overline{E}_4 - E_4^* - 6\varepsilon + 6r - \frac{11}{12}\nu^2}{3}}\right] = 0,$$

$$e^{\nu\,\theta_4 - E_4^*} = e^{-\frac{\overline{E}_4 - E_4^* - 6\varepsilon + 6r - \frac{11}{12}\nu^2}{3}},$$

$$E_4^* - \nu\,\theta_4 = \frac{1}{3}\left(\overline{E}_4 - E_4^* - 6\varepsilon + 6r - \frac{11}{12}\nu^2\right),$$

$$E_4^* = \frac{1}{4}\overline{E}_4 - \frac{3}{2}\left(\varepsilon - r\right) - \frac{11}{48}\nu^2 - \frac{3}{4}\nu\,\theta_4\,. \qquad (43)$$

Then the optimized value function can be written as;

$$V_4^*\left(\overline{E}_4, \alpha_4\right) = 4e^{\alpha_4 - E_4^*} = 4e^{\alpha_4 - \frac{1}{4}\overline{E}_4 + \frac{3}{2}(\varepsilon - r) + \frac{11}{48}\nu^2 + \frac{3}{4}\nu\,\theta_3}\,.$$

(44)

Proceeding such a way we can write the general form of the optimal policy as follows:

$$E_j^* = \frac{1}{j}\overline{E}_j - X_j\nu^2 - \frac{1}{2}(j-1)(\varepsilon - r) + \frac{j-1}{j}\nu\,\theta_j, \tag{45}$$

where $X_j = \frac{j-1}{j}\left(X_{j-1} + \frac{1}{2(j-1)^2}\right)$, for j = 2, 3,...,T and $X_1 = 0$. (46)

In calendar period i for $j = T - i + 1$, (45) and (46) can be written as;

$$E_i^* = \frac{1}{T-i+1}\overline{E}_i - X_i\nu^2 - \frac{1}{2}(T-i)(\varepsilon - r) + \frac{T-i}{T-i+1}\nu\,\theta_i, \text{ and} \tag{47}$$

$X_i = \frac{T-i}{T-i+1}\left(X_{i+1} + \frac{1}{2(T-i)^2}\right)$, for i = 2, 3,...,T-1 and $X_T = 0$. (48)

The generalized form of the optimized value function can be written as;

$$V_j^*\left(\overline{E}_j\right) = je^{\alpha_j - E_j^*} = je^{\alpha_j - \frac{1}{j}\overline{E}_j} + X_j\nu^2 + \frac{1}{2}(j-1)(\varepsilon - r) - \frac{j-1}{j}\nu\,\theta_j. \tag{49}$$

In calendar period i for $j = T - i + 1$, (49) can be written as;

$$V_i^*\left(\overline{E}_i\right) = ie^{\alpha_i - E_i^*} = ie^{\alpha_i - \frac{1}{T-i+1}\overline{E}_i + X_i\nu^2 + \frac{1}{2}(T-i)(\varepsilon - r) - \frac{T-i}{T-i+1}\nu\,\theta_i}. \tag{50}$$

Operating logarithm on (50) we get;

$$\ln\left(p_i^*\right) = \alpha_i - \frac{1}{T-i+1}\overline{E}_i + X_i\nu^2 + \frac{1}{2}(T-i)(\varepsilon - r) - \frac{T-i}{T-i+1}\nu\,\theta_i \tag{51}$$

$$= \alpha_0 + i\varepsilon + \nu\,\theta_i - \frac{1}{T-i+1}\overline{E}_i + X_i\nu^2 + \frac{1}{2}(T-i)(\varepsilon - r) - \frac{T-i}{T-i+1}\nu\,\theta_i$$

$$= \alpha_0 + i\varepsilon - \frac{1}{T-i+1}\overline{E}_i + X_i\nu^2 + \frac{1}{2}(T-i)(\varepsilon - r) - \frac{1}{T-i+1}\nu\,\theta_i. \tag{52}$$

We observe that the optimal emissions policy in (47) is similar to the certainty case in favor of rate share of the remaining allowances $\frac{1}{T-i+1}\overline{E}_i$ and the linear growth factor $\frac{1}{2}(T-i)(\varepsilon - r)$. Also there is a deduction in the current emissions level $X_i\nu^2$, which is tied to the overall volatility of emissions. Again there is the component of emissions that fluctuates with the current realization of costs $\frac{T-i}{T-i+1}\nu\,\theta_i$. If the remaining number of periods is large, then the coefficient is close to

1. This indicates that all of the volatility in the cost parameter is absorbed in adjustment to the current level of emissions which keeps the current level of marginal cost approximately constant. Since the remaining number of periods decline, the coefficient on the quantity adjustment decline. Hence only a portion of the volatility in the cost parameter is absorbed in adjustment to the current level of emissions, because the aggregate emissions are constraint. If there are fewer remaining periods to share the remaining costs, a larger fraction must be absorbed in the current period. As a result the final period shows the price begins to reflect a portion of the volatility of the cost parameter.

2.3 Results in the Permanent Uncertainty Case

If the per period's shock is permanent the general forms are the same as uncertain case and we obtain here the relations are the same as equations (28) to (34). The optimal emissions policy in (29) is identical to the certainty case. But here emissions are completely unresponsive to shocks to the cost parameter. Since none of the cost uncertainty is absorbed by the quantity of emissions, all of the cost uncertainty must be absorbed by the price of (34). It is the best policy since the GHGs are a stock pollutant therefore the optimal policy must be a price control. Weitzman's (Weitzman 1974) original paper was about a policy maker which was uninformed about cost but the producer was informed. Parsonsa and Taschini (2011) show that the volatility of emissions and of the logarithm price one period in temporary uncertainty are as follows:

$$Var_{i-1}\left(E_i^*\right)=\frac{T-i}{T-i+1}\nu \quad \text{and} \quad Var_{i-1}\ln\left(E_i^*\right)=\frac{1}{T-i+1}\nu . \tag{53}$$

But they show the corresponding results in permanent uncertainty case are as follows:

$$Var_{i-1}\left(E_i^*\right)=0 \quad \text{and} \quad Var_{i-1}\ln\left(E_i^*\right)=\nu . \tag{54}$$

The two cases (53) and (54) gives the difference between uncertainties in cost should have upon the cost minimizing emissions path depending upon whether it is a temporary uncertainty or a permanent uncertainty. In the temporary case uncertainty is the quantity of emissions which absorbs shocks to the cost parameter but the price of emissions is relatively constant. In the case of permanent uncertainty the quantity is constant and it is price that absorbs shocks to the cost parameter (Parsonsa and Taschini, 2011). Greenhouse gases are a stock pollutant so that the optimal policy must be a price control which only corresponds with the temporary uncertainty case. In the case of permanent uncertainty, a price control will clearly not be optimal as it is the price that ought to absorb all of the shocks to cost.

Economists always tell casually of a cap and trade system as being a quantity control and a carbon tax a price control. In real life a cap and trade system allows banking and borrowing of allowances across periods can mimic the benefits of a price control. If the cap and trade system faces temporary uncertainty in costs, then it will be the period by period quantity of emissions which will fluctuate under the cap and trade system, and the price will be relatively constant. If the cap and trade system faces permanent uncertainty in costs, then it will be the period by period price which will fluctuate under the cap and trade system, and the quantity of emissions in each period will not be stochastic, but rise deterministically at the rate of growth in costs less the interest rate.

3. CONCLUDING REMARKS

In this paper we have shown stocks and shocks for GHG over price and quantity controls. We have discussed both the certainty and uncertainty in carbon tax or cap and trade system. Recently the global climate change has become the headache of the environment analysts, the economists and the governments of the all nations. Hence there are no alternatives of reductions of greenhouse gases which cause the global warming. Throughout the paper the mathematical calculations are given in some details.

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