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Optimal Ordering and Trade Credit Policy for EOQ Model

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ABSTRACT

Trade credit is the most prevailing economic phenomena used by the suppliers for encouraging the retailers to increase their ordering quantity. In this article, an attempt is made to derive a mathematical model to find optimal credit policy and hence ordering quantity to minimize the cost. Even though, credit period is offered by the supplier, both parties (supplier and retailer) sit together to agree upon the permissible credit for settlement of the accounts by the retailer. A numerical example is given to support the analytical arguments.

JEL. Classification: C02; C61

Key words: Trade Credit, Optimal ordering quantity, Lot-size

1. INTRODUCTION

The classical EOQ model is based on the assumption that the retailer must pay for the items as soon as it is received by the system. However, the most prevailing practice is that the supplier may offer a credit period to the retailer to settle his account within the allowable settlement period. The supplier will vary terms in anticipation of capturing new business, to attract specific group of customers to achieve marketing goals i.e. for supplier who offers trade credit, it is an effective means of price discrimination as well as efficient tool to stimulate the demand of his products.

Haley and Higgins (1973) studied the interaction between inventory policy and trade credit in the context of the classical lot – size model. Goyal (1985) developed mathematical model when supplier offers permissible credit period to settle the account, so that no interest charges are payable from the outstanding amount if the

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account is settled within the allowable delay period. The supplier will obviously charge higher interest if the account is not settled by the end of the permissible credit. In fact, this brings some economic advantage to the system as retailer would try to earn some interest from the revenue generated during the period of the permissible delay. Shah et al. (1988) extended above model by allowing shortages. Mandal and Phaujdar (1989) included interest earned from the sales revenue on the stock remaining beyond the settlement period. Chung and Huang (2003) extended Goyal's model for finite replenishment rate. Related research articles are by Davis and Gaither (1985), Arcelus and Srinivasan (1993), Shah (1993), Aggarwal and Jaggi (1995), Hwang and Shinn (1997), Jamal et al. (1997), Shah et al. (1997), Shinn (1997), Chu et al. (1998), Chung (1998), Shah and Shah (1998), Chang and Dye (2000), Chung (2000), Jamal et al. (2000), Chung et al. (2001), Sarker et al. (2001), Abad and Jaggi (2003), Chang et al. (2003), Gor and Shah (2003), Shinn and Hwang (2003), Chung and Liao (2004), Shah (2004), Shah et al. (2004), Chung et al. (2005), Gor and Shah (2005 a, 2005 b), Lokhandwala et al.(2005), Ouyang et al. (2005), Shah and Trivedi (2005), Teng et al. (2005) and Yang and Wee (2006) etc.

The above stated articles assumed that trade credit is constant even though most of the articles stated that allowable trade credit can be considered as demand increasing phenomena. In this article, an attempt is made to derive optimal trade credit and ordering policy for the retailer to minimize the total cost of the inventory system. It is established that the total cost per time unit of an inventory system is a function of credit period. The analytical results are supported by a numerical example.

2. ASSUMPTIONS AND NOTATIONS

The following assumptions are used to aforesaid model:

1. The inventory system deals with the single item.
2. The demand is $R = \alpha M^{\beta}$; $\alpha, \beta > 0, \alpha$? β, M denotes trade credit offered by the supplier and a decision variable.
3. Shortages are not allowed and lead time is zero.
4. Replenishment is instantaneous.
5. Replenishment rate is infinite.
6. If the retailer pays by M, then supplier does not charge any interest. If the retailer pays after M, he can keep the difference in the unit sale price and unit cost in an interest bearing account at the rate of Ie/unit/year.

The notations are as under:

h	=	The inventory holding cost/unit/year excluding interest charges.
p	=	The selling price/unit.
C	=	The unit purchase cost, with C < p.
A	=	The ordering cost/order.
M	=	The credit period in settling the account. (a decision variable)
T	=	The replenish cycle time (a decision variable)
Ic	=	The interest charged per $ in stock per year by the supplier when retailer pays during [M, T].

Ie	=	The interest earned/$/year.
IHC	=	Inventory holding cost/time unit.
PC	=	Purchase cost / time unit.
OC	=	Ordering cost / time unit.
IE	=	Interest earned / time unit.
IC	=	Interest charged / time unit.
(t)	=	The on-hand inventory level at time t ($0 \le t \le T$).
$K_i(T)$	=	The total cost of an inventory system per time unit, i = 1, 2.

3. MATHEMATICAL FORMULATION

The on-hand inventory depletes due to demand R(M). The instantaneous state of inventory at any instant of time t, $0 \le t \le T$ is governed by the differential equation

$$\frac{dQ(t)}{dt} = -R(M) = -\alpha M^{\beta}, \; 0 \le t \le T \qquad (3.1)$$

with initial condition Q(0) = Q and boundary condition Q(T) = 0. Consequently, the solution of (3.1) is given by

$$Q(t) = R(M)(T - t); \quad 0 \le t \le T \qquad (3.2)$$

and the order quantity is $Q = R(M)T$ (3.3)

The cost components per unit time are as follows:

- Ordering cost; OC = A/T (3.4)
- Inventory holding cost;

$$IHC = \frac{h}{T}\int_0^T Q(t)dt = \frac{hR(M)T}{2} \qquad (3.5)$$

Regarding interest charged and earned, based on the length of the cycle time T, two cases arise:

Case 1: $T \le M$

Case 2: $M < T$

We discuss each case in detail.

Case 1: $T \le M$

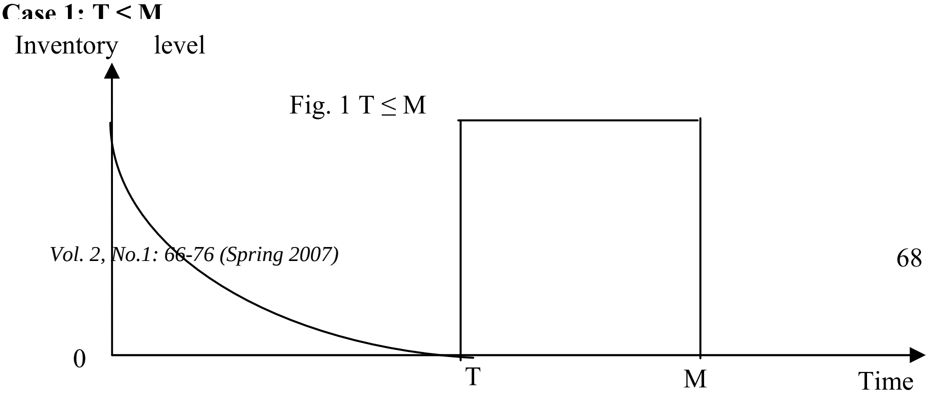


Fig. 1 $T \le M$

Here, the retailer sells Q-units during [0, T] and is paying CR(M)T in full to the supplier at time M $\geq$ T. So interest charges are zero. i.e. $IC_1 = 0$.
(3.6)

The retailer sells products during [0, T] and deposits the revenue in an interest bearing account at the rate of I_e/\$/year. In the period, [T, M] the retailer deposits revenue into the account that earns I_e/\$/year. Therefore total interest earned per time unit is

$$IE_1 = \frac{pIe}{T}\left[\int_0^T R(M).t\ dt + R(M)T(M-T)\right] = \frac{p\ Ie\ R(M)(2M-T)}{2}$$

(3.7)

Hence, the total cost; $K_1(T,M)$ per time unit of an inventory system is given by

$$K_1(T,M) = OC + IHC + IC_1 - IE_1$$

(3.8)

Here, T and M are continuous decision variables. The optimal values of M and T can be obtained by solving

$$\frac{\partial K_1(T,M)}{\partial M} = \frac{hT\alpha M^{\beta}\beta}{2} - \frac{p\ Ie\ \alpha M^{\beta-1}\beta(2M-T)}{2} - p\ Ie\ \alpha M^{\beta} = 0 \quad (3.9\ a)$$

$$\frac{\partial K_1(T,M)}{\partial T} = -\frac{A}{T^2} + \frac{(h+p\ Ie)\alpha M^{\beta}}{2} = 0$$

(3.9 b)

simultaneously by suitable numerical method.

The obtained M and T minimizes the total cost $K_1(T,M)$; provided

$XY - Z^2 > 0$,

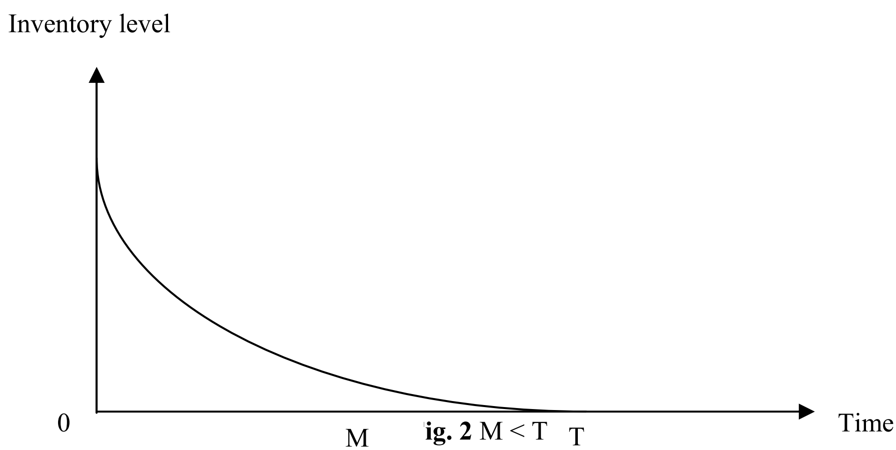
(3.9 c)

where $X = \frac{\partial^2 K_1(T,M)}{\partial^2 T} = \frac{2A}{T^2}$,

$$Z = \frac{\partial^2 K_1(T,M)}{\partial M \partial T} = \frac{(h+p\ Ie)\alpha M^{\beta-1}\beta}{2}$$

$$Y = \frac{\partial^2 K_1(T,M)}{\partial M^2} = \frac{hT\alpha M^{\beta-2}\beta(\beta-1)}{2} - \frac{p\ Ie\ \alpha M^{\beta-2}\beta(\beta-1)(2M-T)}{2} - 2p\ Ie\ \alpha M^{\beta-1}\beta$$

Case 2: M < T < N



ig. 2 M < T

The retailer sells units at selling price *p* \$/unit and deposits the revenue into an interest bearing account at an interest rate Ie/unit/annum during [0, M]. Therefore, interest earned during [0, M] is given by

$$IE_2 = \frac{pIe}{T}\int_0^M R(M).t\ dt = \frac{p\ Ie\ \alpha\ M^{\beta+2}}{2T} \quad (3.10)$$

and during [M, T] supplier will charge interest rate at Ic/unit/annum. So total interest charged per time unit during [M, T] is

$$IC_2 = \frac{CIc}{T}\int_M^T Q(t)dt = \frac{CIcR(M)(T-M)^2}{2T} \quad (3.11)$$

The total cost; $K_2(T,M)$ per time unit of an inventory system is given by

$$K_2(T,M) = OC + IHC + IC_2 - IE_2 \quad (3.12)$$

The optimal values of M and T can be obtained by simultaneously solving

$$\frac{\partial K_2(T,M)}{\partial M}=\frac{h\ T\ \ \alpha M^{\beta-1}\beta}{2}+\frac{C\ Ic\ \ \alpha M^{\beta-1}\beta(T-M)^2}{2T}-\frac{C\ Ic\ \ \alpha M^{\beta}(T-M)}{T}$$

$$-\frac{p\ Ie\ \ \alpha M^{\beta+1}}{2T}\left(\beta+2\right)=0$$

(3.12 a)

$$\frac{\partial K_2(T,M)}{\partial T}=-\frac{A}{T^2}+\frac{h\ \alpha M^{\beta}}{2}+\frac{C\ Ic\ \ \alpha M^{\beta}(T^2-M^2)}{2T^2}+\frac{p\ Ie\ \ \alpha M^{\beta+2}}{2T^2}=0$$

(3.12 b)

The obtained M and T minimizes the total cost $K_2(T,M)$; provided

$$EF-G^2>0$$

(3.12 c)

where,

$$\text{E}=\frac{\partial^2 K_2(T,M)}{\partial T^2}=\frac{2A}{T^3}+\frac{C\ Ic\ \ \alpha M^{\beta}}{T}-\frac{C\ Ic\ \ \alpha M^{\beta}(T^2-M^2)}{T^3}-\frac{p\ Ie\ \ \alpha M^{\beta+2}}{T^3}$$

$$F=\frac{\partial^2 K_2(T,M)}{\partial M^2}=\frac{h\ T\ \ \alpha M^{\beta-2}\beta(\beta-1)}{2}+\frac{C\ Ic\ \ \alpha M^{\beta-2}\beta(\beta-1)(T-M)^2}{2T}$$

$$-\frac{2C\ Ic\ \ \alpha M^{\beta-1}\beta(T-M)}{T}+\frac{C\ Ic\ \ \alpha M^{\beta}}{T}-\frac{p\ Ie\ \ \alpha M^{\beta}}{2T}\left(\beta+1\right)\left(\beta+2\right)$$

$$G=\frac{\partial^2 K_2(T,M)}{\partial M\partial T}=\frac{h\ \alpha M^{\beta-1}\beta}{2}+\frac{C\ Ic\ \ \alpha M^{\beta-1}\beta(T^2-M^2)}{2T^2}-\frac{C\ Ic\ \ \alpha M^{\beta+1}}{T^2}$$

$$+\frac{p\ Ie\ \ \alpha M^{\beta+1}}{2T^2}\left(\beta+2\right)$$

4. SOME RESULTS

Proposition 4.1: $K_i(T,M)$ is decreasing function of M (i = 1, 2).

Proof: Clearly,

$$\frac{\partial K_1(T,M)}{\partial M}=\frac{hT\alpha M^{\beta}\beta}{2}-\frac{p\ Ie\ \ \alpha M^{\beta-1}\beta(2M-T)}{2}-p\ Ie\ \ \alpha M^{\beta}<0$$

$$\frac{\partial K_2(T,M)}{\partial M}=\frac{h\ T\ \alpha M^{\beta-1}\beta}{2}+\frac{C\ Ic\ \alpha M^{\beta-1}\beta(T-M)^2}{2T}-\frac{C\ Ic\ \alpha M^{\beta}(T-M)}{T}$$
$$-\frac{p\ Ie\ \alpha M^{\beta+1}}{2T}\left(\beta+2\right)<0$$

Proposition 4.2: $K_i(T,M)$ is deceasing function of T (i = 1, 2).

Proof: Clearly,

$$\frac{\partial K_1(T,M)}{\partial T}=-\frac{A}{T^2}+\frac{(h+p\ Ie)\alpha M^{\beta}}{2}<0$$

$$\frac{\partial K_2(T,M)}{\partial T}=-\frac{A}{T^2}+\frac{h\ \alpha M^{\beta}}{2}+\frac{C\ Ic\ \alpha M^{\beta}(T^2-M^2)}{2T^2}$$
$$+\frac{p\ Ie\ \alpha M^{\beta+2}}{2T^2}<0$$

Proposition 4.3: $K_i(T,M)$ is increasing function of α (i = 1, 2).

Proof: Clearly,

$$\frac{\partial K_1(T,M)}{\partial \alpha}=\frac{h\ T\ M^{\beta}}{2}-\frac{p\ Ie\ M^{\beta}(2M-T)}{2}>0$$

$$\frac{\partial K_2(T,M)}{\partial \alpha}=\frac{h\ T\ M^{\beta}}{2}+\frac{C\ Ic\ M^{\beta}(T-M)^2}{2T}-\frac{p\ Ie\ M^{\beta+2}}{2T}>0$$

Proposition 4.4: $K_i(T,M)$ is decreasing function of β (i = 1, 2).

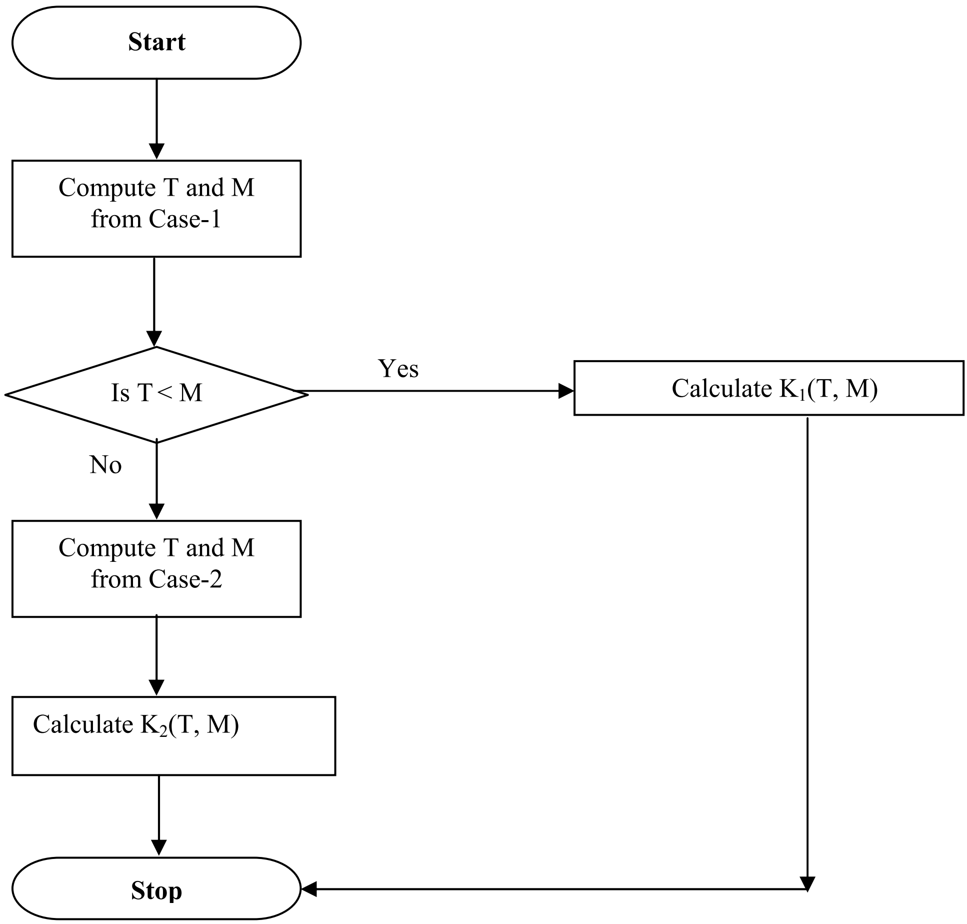
Proof: Clearly,

$$\frac{\partial K_1(T,M)}{\partial \beta}=\frac{\alpha\ M^{\beta}\ln(M)}{2}\left(h\ T-p\ Ie\ (2M-T)\right)<0$$

$$\frac{\partial K_2(T,M)}{\partial \beta}=\frac{\alpha\ M^{\beta}\ln(M)}{2T}\left(h\ T^2-C\ Ic\ (T-M)^2-p\ Ie\ M^2\right)<0$$

In the next section, computation flow chart is given to search for optimal solution.

5. FLOW CHART


Start
Compute T and M from Case-1
Is T < M
Yes
Calculate K_1(T, M)
No
Compute T and M from Case-2
Calculate K_2(T, M)
Stop

6. NUMERICAL EXAMPLE

Consider following parametric values:

[A, C, h, Ie, p, Ic] = [200, 20, 2, 0.12, 30, 0.18]

β ↓ \ α →	1000	1500	2000
0.1	T = 0.3229 M = 0.0228 R = 685.25 *Q* = 221.24 $K_2(T,M)$ = 1172.62	T = 0.2662 M = 0.0188 R = 1008.22 *Q* = 268.36 $K_2(T,M)$ = 1434.49	T = 0.2321 M = 0.0164 R = 1326.00 *Q* = 307.76 $K_2(T,M)$ = 1645.10
0.2	T = 0.3628 M = 0.0470 R = 542.60 *Q* = 196.87 $K_2(T,M)$ = 1010.59	T = 0.3018 M = 0.0391 R = 784.45 *Q* = 236.71 $K_2(T,M)$ = 1215.11	T = 0.2648 M = 0.0343 R = 1018.93 *Q* = 269.78 $K_2(T,M)$ = 1384.86
0.3	T = 0.3972 M = 0.0713 R = 452.80 *Q* = 179.84 $K_2(T,M)$ = 890.90	T = 0.3330 M = 0.0598 R = 644.21 *Q* = 214.51 $K_2(T,M)$ = 1062.66	T = 0.2938 M = 0.0527 R = 827.31 *Q* = 243.09 $K_2(T,M)$ = 1204.24

7. CONCLUSION

In this paper, an attempt is made to develop an EOQ model in which demand is assumed to be increasing function of credit period (a decision variable) when supplier offers a credit period, if retailer could not settle his account. Increase in fixed partial demand decreases trade credit and increases annual demand significantly. Exponent increase in demand increases trade credit and decreases annual demand significantly. An easy – to – use computational flow-chart is given to search for optimal policy. The observed managerial issues are as follows:

(1) .Increase in fixed partial demand increases the order quantity and total cost of an inventory system.

(2) Increase in exponent factor β decreases the order quantity and total cost of an inventory system.

REFERENCES

Abad, P. L and C. K Jaggi. 2003. A Joint Approach for Setting Unit Price and the Length of the Credit Period for a Seller When End Demand Is Price Sensitive. *International Journal of Production Economics*, 83(2): 115-122.

Aggarwal, S. P. and C. K. Jaggi. 1995. Ordering Policies of Deteriorating Items under Permissible Delay in Payments. *Journal of Operational Research Society*, 46(5): 658 – 662.

Arcelus, F. J. and G. Srinivasan. 1993. Delay of Payments for Extra Ordinary Purchases. *Journal of Operational Research Society*, 44: 785 – 795.

Chang H. J. and C. Dye. Y.2000. An Inventory Model for Deteriorating Items With Partial Backlogging and Permissible Delay in Payments. *International Journal of Systems Science*, 32: 345–52.

Chang, C. T., L. Y. Ouyang and J.T. Teng. 2003. An EOQ Model for Deteriorating Items Under Supplier Credits Linked to Ordering Quantity. *Applied Mathematical Modeling*, 27: 983 – 996.

Chu, P., K.J. Chung and S.P. Lan. 1998. Economic Order Quantity of Deteriorating Items under Permissible Delay in Payments. *Computers and Operations Research*, 25: 817 – 824.

Chung, K. J. 1998. A Theorem on the Determination of Economic Order Quantity under Conditions of Permissible Delay in Payments. *Computers and Operations Research*, 25: 49 – 52.

Chung, K. J. 2000. The Inventory Replenishment Policy for Deteriorating Items under Permissible Delay in Payments. *Opsearch,* 37: 267 – 281.

Chung, K. J. and Y.F. Huang. 2003. The Optimal Cycle Time for EPQ Inventory Model under Permissible Delay in Payments. *International Journal of Production Economics*, 84 (3): 307-318.

Chung, K. J. and J.J. Liao. 2004. Lot-Sizing Decisions under Trade Credit Depending On the Ordering Quantity. *Computers & Operations Research*, 31 (6): 909-928.

Chung, H. J., C.H. Hung and C.Y. Dye. 2001. An Inventory Model for Deteriorating Items with Linear Trend Demand under the Condition of Permissible Delay in Payments. *Production Planning and Control*, 12: 274–82.

Chung, K. J., S.K. Goyal and Yung-Fu. Huang. 2005. The Optimal Inventory Policies under Permissible Delay in Payments Depending On the Ordering Quantity. *International Journal of Production Economics*, 95 (2): 203-213.

Davis, R. A. and N. Gaither. 1985. Optimal Ordering Policies under Conditions of Extended Payment Privileges. *Management Science*, 31: 499 – 509.

Gor, Ravi and Nita H. Shah. 2003. An Order - Level Lot - Size Model with Time Dependent Deterioration and Permissible Delay in Payments. *Advances and Applications in Statistics,* 3 (2): 159 – 172.

Gor, Ravi and Nita H. Shah. 2005. An EOQ Model for Deteriorating Items with Two Parameters Weibull Distribution Deterioration under Supplier Credits. *Journal of Mathematics and Systems Sciences*, 1 (1): 2 – 18.

Gor, Ravi and Nita H. Shah. 2005. A Lot – Size Model with Variable Deterioration Rate under Supplier Credits. *Applied Mathematical Analysis and Application*, 1 (1): 65 – 76.

Goyal, S. K. 1985. Economic Order Quantity under Conditions Of Permissible Delay in Ayments. *Journal of Operational Research Society*, 36: 335 – 338.

Haley, C. W. and R.C. Higgins. 1973. Inventory Policy and Trade Credit Financing. *Management Science*, 20: 464 – 471.

Hwang, H. and S. W. Shinn. 1997. Retailer's Pricing and Lot Sizing Policy for Exponentially Deteriorating Products under the Condition of Permissible Delay In Payments. *Computers and Operations Research*, 24: 539 – 547.

Jamal, A. M., B. R. Sarker and S. Wang. 1997. An ordering policy for deteriorating items with allowable shortages and permissible delay in payment, *Journal of Operational Research Society*, 48: 826 – 833.

Jamal, A. M., B.R. Sarker and S. Wang. 2000. Optimal payment time for a retailer under permitted delay of payment by the wholesaler, *International Journal of Production Economics*, 66 (1): 59 – 66.

Lokhandwala, K., Nita H. Shah and Y.K. Shah. 2005. Optimal Ordering Policies under Conditions of Extended Payment Privileges for Deteriorating Items. *Revista Investigation Operational (Cuba)*, 26(3): 1 – 8.

Mandal, B. N. and S. Phaujdar. 1989a. Some EOQ Models under Permissible Delay in Payments. *International Journal of Management and Systems*, 5 (2): 99 – 108.

Mandal, B. N. and S. Phaujdar. 1989-b. An Inventory Model for Deteriorating Items and Stock Dependent Consumption Rate. *Journal of Operational Research Society*, 40: 483 – 488.

Ouyang, L. Y., J.T. Teng, K.W. Chuang. and B.R. Chuang. 2005. Optimal Inventory Policy with Non Instantaneous Receipt under Trade Credit. *International Journal of Production Economics*, 98(3): 290-300.

Sarker, B. R., A.M. Jamal and S. Wang. 2001. Optimal Payment Time under Permissible Delay for Production with Deterioration. *Production planning & Control*, 11: 380 – 390.

Shah, Nita H. 1993a. A Lot Size Model for Exponentially Decaying Inventory When Delay in Payments Is Permissible. *Cahiers du CERO (Belgium)*, 35 (1):1 – 9.

Shah, Nita H. 2004. Probabilistic Order Level System When Items in Inventory Deteriorate and Delay an Payments Is Permissible. *Asia - Pacific Journal of Operational Research, Singapore*, 21 (3): 319 – 331.

Shah, Nita H. and Y.K. Shah. 1998. A Discrete – In – Time Probabilistic Time Scheduling Model for Deteriorating Items under Conditions of Permissible Delay in Payments. *International Journal of System Sciences (UK)*, 29 (2):121 – 125.

Shah, Nita H. and C. J. Trivedi. 2005. An EOQ Model for Deteriorating Items under Permissible Delay in Payments When Supply Is Random. *Revista Investigation Operational (Cuba)*, 26(2):157 – 168.

Shah, V. R., H.C. Patel and Y.K. Shah. 1988. Economic Ordering Quantity When Delay in Payments of Order and Shortages are Permitted. *Gujarat Statistical Review*, 15 (2): 51 – 56.

Shah, Nita H., S.B. Bhavsar and Shah. 1997. (T_j, S_j) – Policy With Increasing Demand When Delay in Payments Is Permissible. *Research Bulletin (Science) of Punjab University*, 47 (1 – 4): 115 – 121.

Shinn, S. W. 1997. Determining Optimal Retail Price and Lot Size under Day-Terms Supplier Credit. *Computers & Industrial Engineering,* 33 (3-4): 717-720.

Shinn, S. W. and H. Hwang. 2003. Retailer's Pricing and Lot Sizing Policy for Exponentially Deteriorating Products under the Condition of Permissible Delay in Payments. *Computers and Industrial Engineering*, 24 (6): 539-547.

Teng, J. T., C.T. Chang and S.K. Goyal. 2005. Optimal Pricing and Ordering Policy under Permissible Delay in Payments. *International Journal of Production Economics*, 97 (2):121-129.

Yang, P. C. and H.M. Wee. 2006. A Collaborative Inventory System with Permissible Delay in Payment for Deteriorating Items. *Mathematical and Computer Modeling*, 43 (3-4): 209-221.

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