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Scaling Aspects of Lyari River Flow Routing

Rana Khalid Naeem* and Asif Mansoor**

ABSTRACT

In this communication we utilize an improved version for model proposed by Manning for the waste flow via an open channel Lyari. We have computed the status of discharges, storages and depth for all locations of the Lyari waste flow using this proposed model. We have found that the result obtained using this model is good agreement for the recent data sets.

JEL Classification: O21; O22; Q22; Q24; Q25; R14;R15

Key words: Open Channel, Proposed Model, Flow, Storage, Depth, Comparison

1. INTRODUCTION

Chezy (1776) developed an equation, which computes mean velocity in an open channel flow and that equation is,

$$V = C\sqrt{RS} \qquad (1)$$

where R is the hydraulic radius in meters and S is the channel slope. In equation (1), C is called Chezy's coefficient and is dimensional, having dimensions $(\text{length})^{1/2}$ per time. The Chezy's coefficient is determined by experiments. Manning performed series of experiments and found that dependence on hydraulic radius is actually not as given in equation (1), and modified equation (1). The modified equation, called Manning equation, is

$$V = \frac{1}{n} R^{2/3} \sqrt{S} \qquad (2)$$

in which *n* is Manning's resistance coefficients and its value dependents on the surface material of the Channel's, wetted perimeter and is determined from experiments. This formula is more accurate then Chezy and is widely being used now a days. The relationship between Chezy's C and Manning's *n* is easily shown to be

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* Prof. Dr. Rana Khalid Naeem is Professor of Mathematics at University of Karachi, Pakistan. E-mail: rknwindsor@yahoo.com

** Asif Mansoor is Research Fellow at University of Karachi, Pakistan.E-mail: asifmansure@yahoo.com

$$C = \frac{1}{n} R^{1/6} \qquad (3)$$

After the appearance of the Manning work, many formulae are proposed for C by researchers; here we quote some of them like: (Kutter 1869 in Yen 2004; Lacey 1929; Chow 1964; Chitale 1966; Diplas 1990; Strupczweski 1990; Sharma 1990; Choudhary 1993; Naot 1996)

Bazin (1897) considered Chezy C to be a function of *R* but not of *S* and presented the following formula for computation of C

$$C = \frac{87}{1 + \frac{M}{\sqrt{R}}} \qquad (4)$$

in which *M* is the coefficient of roughness of the channel.

Pavlovsky (1925) in (Sharma 1990) modified C in Manning formula. The modified C is

$$C = \frac{1}{n} R^{y} \qquad (5)$$

where $y = 2.5\sqrt{n} - 0.13 - 0.75\sqrt{R}(\sqrt{n} - 0.10)$. It depends upon the roughness coefficient (*n*) and hydraulic radius (*R*). The formula is widely used in U.S.S.R.

Kennedy (1895) gave a classic empirical equation correlating mean velocity V in a regime canal with vertical depth *D*, measured on the approximately horizontal silted bed. The mean velocity involves critical velocity V_o given by

$$V_o = 0.55\, D^{0.64} \qquad (6)$$

The hydraulic diagrams given by Kennedy are extensively used in India for designing irrigation canals. These diagrams enable to design any number of channel section for given slope with different bed width and depth.

2. PROPOSED MODEL

In the present work we propose the following formula for computing the mean velocity through an open channel. The formula is:

$$V = \frac{1}{n} R^{\frac{2}{3}} S^{\frac{1}{2}} C_{model} \qquad (7)$$

This formula is an improvement to Manning formula and C_{model} is a dimensionless number defined in (13). It is worth noting that in the proposed *V*, the C_{model} is non-dimensional while in the earlier works in which Chezy C is modified, are all dimensional. The formula is employed in computing *V* through the Lyari River. The computed *V* is compared with the observed values and the values obtained through Manning formula. It is seen that the computed values are in good agreement with the observed values.

3. FLOW ROUTING MODEL

The discharge *Q* through cross sectional area A is given by (Chow 1959; Ponce 1994)

$$Q = AV \qquad (8)$$

where V is the mean velocity of flow.

The Lyari River can be approximated as an open channel with trapezoidal cross section. The mean velocity for flow through it, employing Manning formula, is given by (Souders 2002; Vehoeven 2003)

$$V = \frac{h^{\frac{2}{3}} W^{\frac{2}{3}} \sqrt{s}}{n\left[W + zh\sqrt{1+z^2}\right]^{\frac{2}{3}}} \qquad (9)$$

where *W*, *h* and *z* are river width, channel flow depth and side slope respectively.

Since the cross sectional area *A* is given by

$$A = Wh \qquad (10)$$

The equation (8), utilizing equation (9) and equation (10), yields

$$Q_{manning} = \frac{W^{\frac{5}{3}} h^{\frac{5}{3}} \sqrt{s}}{n\left[W + zh\sqrt{1+z^2}\right]^{\frac{2}{3}}} \qquad (11)$$

In order to obtain dimensionless expression for *C* we set

$$C = C(R, W) \qquad (12)$$

The dimensional analysis indicates that *C* can be expressed in terms of π-term '*R/W*'' and, therefore, we write .

$$C_{model} = C_{model}(R/W) \qquad (13)$$

Many rational approximations for C_{model} are attempted by others and it is found that ratio of two cubics for C_{model} produces very good results. The rational approximation for C_{model} is

$$C_{model} = \frac{1 + ax + bx^2 + cx^3}{1 + dx + ex^2 + fx^3} \qquad (14)$$

where $x = R/W$ and

a = 1.09E-3, b = 8.29E-3, c = 1.08E-3, d = 8.99E-4, e = 1.17E-3, f = 9.15E-4

The discharge using C_{model} is

$$Q_{model} = \left(\frac{1 + .00109x + .00829x^2 + .00108x^3}{1 + .000899x + .00117x^2 + .000915x^3}\right)\left[\frac{W^{\frac{5}{3}} h^{\frac{5}{3}} \sqrt{s}}{n\left[W + zh\sqrt{1+z^2}\right]^{\frac{2}{3}}}\right] \qquad (15)$$

The balance of the surface water store, *S*, within two river bridges is given by (Kraijenhoff and Moll 1986)

$$\frac{dS}{dt} = I - Q \qquad (16)$$

in which *I* is the inflow

The surface water, *S*, is assumed to be a linear function of outflow discharge (Hudson and Hazen 1964), so

$$S = \tau Q \qquad (17)$$

in which 'τ' is the travel time between two bridges under consideration. If 'L' is the distance between two bridges, then

$$\tau = \frac{L}{V} \tag{18}$$

Equation (16), utilizing equation (8) and (17), yields

$$\frac{dh}{dt} = \frac{1}{LW}\left[I - WhV\right] \tag{19}$$

Equation (19) describes the flow in terms of rate of change of the flow depth for a given river section.

On using an explicit forward step finite difference approximation for equation (19), we obtain

$$h_{t+1} = h_t + \frac{\Delta t}{LW}\left[I_t - WhV_t\right] \tag{20}$$

Equation (16) is solved numerically. The discretization of equation (19) on *xt*-plane (Figure 1) leads to

$$\frac{I_t + I_{t+1}}{2} - \frac{Q_t + Q_{t+1}}{2} = \frac{S_{t+1} - S_t}{2} \tag{21}$$

in which

I_t = inflow at time level 1, I_{t+1} = inflow at time level 2, Q_t = outflow at time level 1, Q_{t+1} = outflow at time level 2, S_t = storage at time level 1, S_{t+1} = storage at time level 2

Δt = time interval

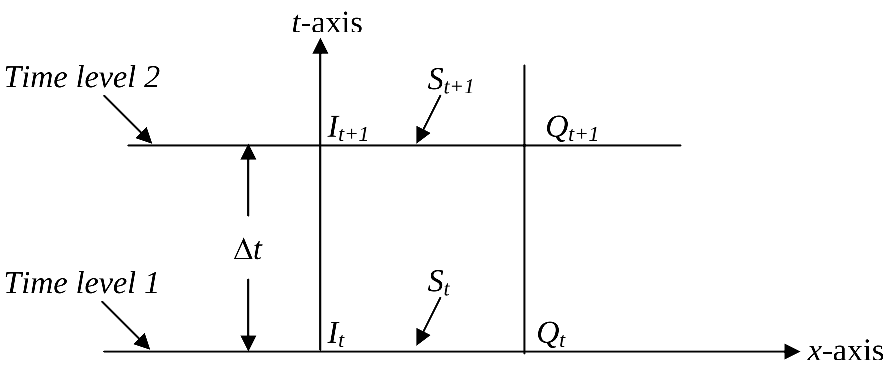


Figure-1. Discretization of Storage Equation in *xt*-plane

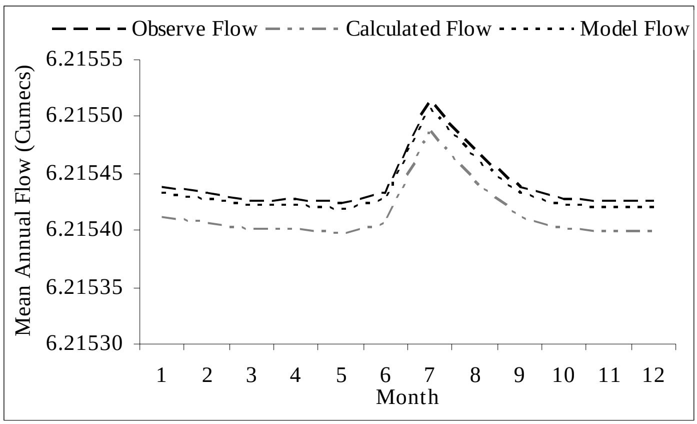
Then Storage (S_{t+1}) at any time 't+1', after rearranging equation (20), is given by

$$S_{t+1} = S_t + \frac{1}{2}\left[I_t + I_{t+1}\right]\Delta t - \frac{1}{2}\left[Q_t + Q_{t+1}\right]\Delta t \tag{22}$$

4. COMPARISON OF COMPUTED AND OBSERVED VALUES

Comparison are made between discharges (mean monthly, mean annual), mean monthly storage, mean monthly depth obtained from Manning and proposed C_{model} with each other and observation at all eleven bridges of the Lyrai River it is seen that at all locations of the Lyari River C_{model} produces better results than the Manning formula. For the sake of convenience we can give only graphs for a few locations of the Lyari River. These are depicted in figures (2-5).

Figure-2. Comparison of Mean Annual Discharges



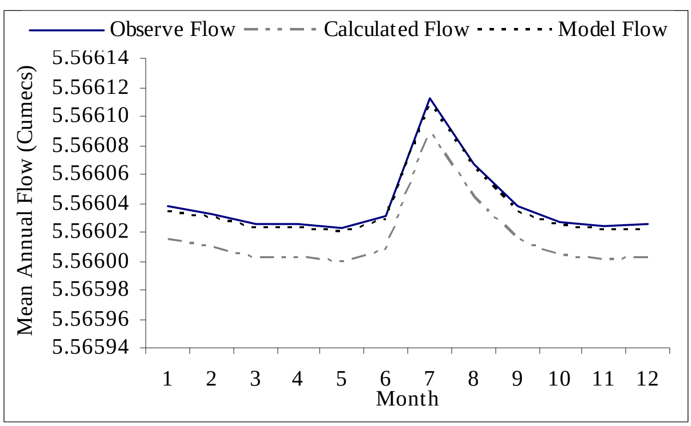
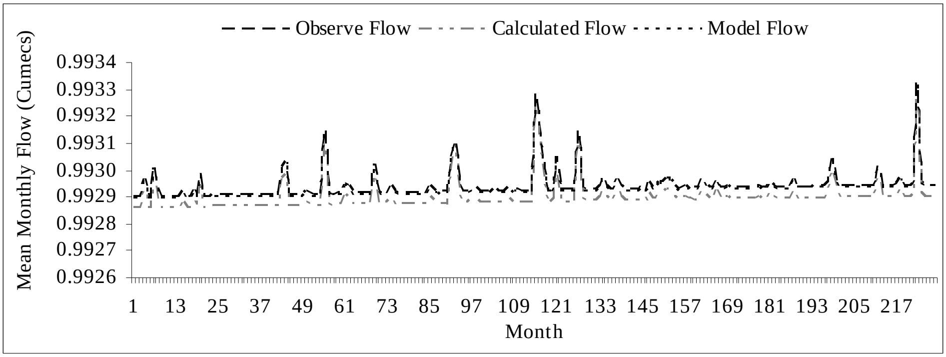


Figure-3. Comparison of Mean Monthly Discharges



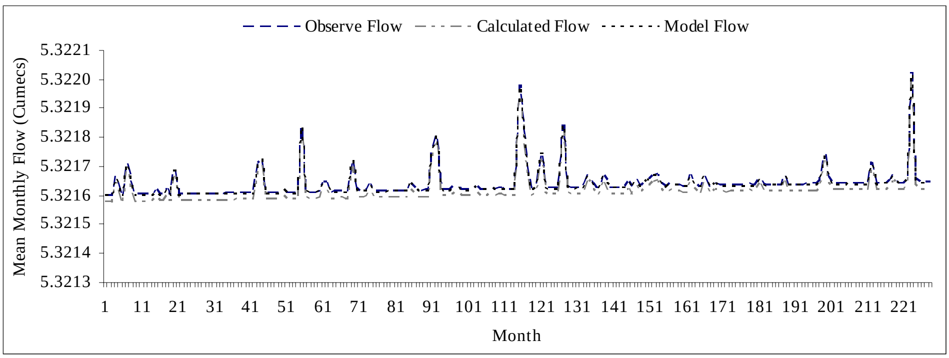
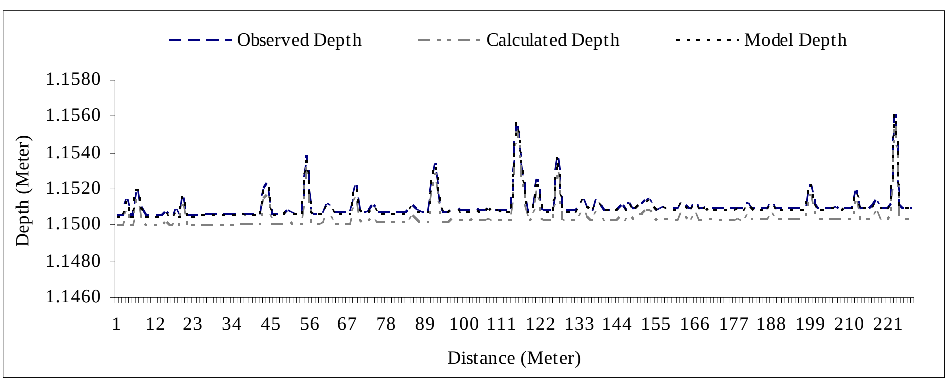


Figure-4. Comparison of Depths at Different Locations of River Lyari



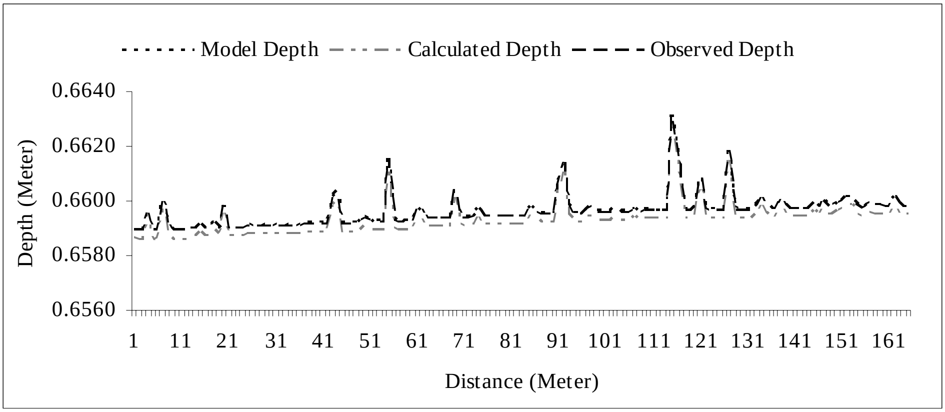
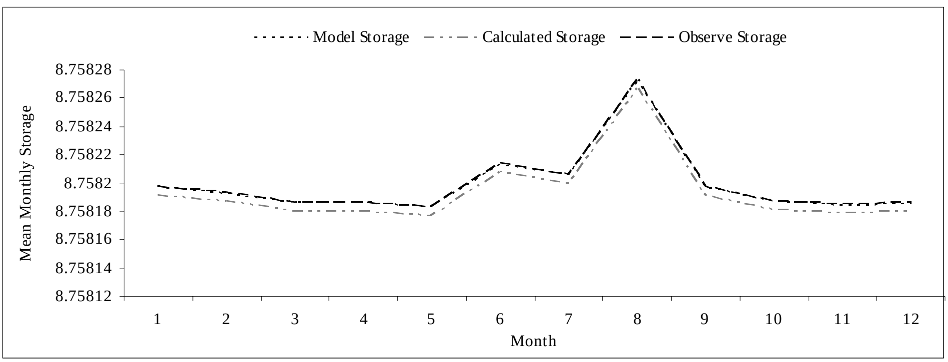


Figure 5. Comparison of mean monthly storage between two consecutive bridges.



4. CONCLUSION

In this work an expression for C_{model} is proposed for the computation of mean flow velocity *V* through an open channel. The discharges, storages and depths are computed using the proposed C_{model}. These are compared with using Manning formula and the observed formula, and it is seen that results obtained through proposed C_{model} are much closer to the observed values. This indicates that C_{model} is an improvement over the Manning formula.

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